Finding Nash Equilibria

Suppose you want to find all the Nash equilibria in the following game:

		Guy	
		Ballet	Hockey
Girl	Ballet	3,1	0,0
	Hockey	0,0	1,4

Obviously, (Ballet, Ballet) and (Hockey, Hockey) are the pure-strategy Nash equilibria. But does there exist one where the players randomize (*i.e.* play a strategy that is not pure, also called "mix")?

First, convince yourself that in this game, if a player plays a pure strategy, then the other player's best response is a pure strategy as well. This allows you to conclude that in any non-pure NE, **both** players must randomize. As we've explained in class, this means that both players are **indifferent between their actions**. (Note: this is not true for all games - in some 2x2 games, there are NE where only one player mixes.)

Let's now find the probabilities with which the players randomize.

1. Let p be the probability that the Girl plays Ballet. Then the Guy's expected utility from Ballet is: $p(1) + (1-p)(0)$, while his expected utility from Hockey is: $p(0) + (1-p)(4)$. Therefore, for the Guy to be indifferent, we must have $p(1) + (1-p)(0) = p(0) + (1-p)(4)$, or $p = 4 - 4p$, which implies $p = 0.8$.

2. Let q be the probability that the Guy plays Ballet. Then the Girl's expected utility from Ballet is: $q(3) + (1-q)(0)$, while her expected utility from Hockey is: $q(0) + (1-q)(1)$. Therefore, for the Girl to be indifferent, we must have $q(3) + (1-q)(0) = q(0) + (1-q)(1)$, or $3q = 1 - q$, which implies $q = 0.25$.

Therefore, the Nash equilibrium where players randomize is:

(0.8 Ballet and 0.2 Hockey, 0.25 Ballet and 0.75 Hockey)